Math 3130 Test 1 100 points

Name Chapter 7 Fall 2006 Score:

Part I (20points): Circle the letter that corresponds to the right answer.

1. The existence of the Laplace transform of a function $f(t)$ is guaranteed provided

A. $f(t)$ is piecewise continuous on $[0,\infty)$.

B. $f(t)$ is of exponential order.

C. either A or B

D. both A and B

2. If the Laplace transform $F(s)$ of a function $f(t)$ exists, then

A. $\lim_{s\to\infty} F(s) = 0$

B. $\lim_{s\to\infty} F(s) = \infty$

C. $\lim_{s\to 0} F(s) = 0$

D. $\lim_{s\to 0} F(s) = \infty$

3. The function $f(t) = 4e^{3t}\cos 2t$ is of exponential order

A. 1

B. 2

C. 3

D. 4

4. The integral $\int_0^t (t-\tau)^3 \cos\tau d\tau$ is the as

A. $\int_0^t t^3 \cos(t-\tau) d\tau$

B. $\int_0^t \tau \cos(t-\tau)^3 d\tau$

C. $\int_0^t \tau^3 \cos(t-\tau) d\tau$

D. $\int_0^t t^3 \cos\tau d\tau$

5. The Laplace transform of the function $t^4 f(t)$ requires finding

A. the 3rd derivative of $f(t)$.

B. the 4rd derivative of $f(t)$.

C. the 3rd derivative of $F(s)$.

D. the 4rd derivative of $F(s)$.

Part II (20points): Circle true or false.

6. True or False: If f is not piecewise continuous on $[0,\infty)$, then the Laplace transform of f (t) will not exists.

7. True or False: The function $f(t) = (e^t)^{20}$ is not of exponential order.

8. True or False: $F(s) = \dfrac{s}{s^2 + 25}$ is not the Laplace transform of a function that is piecewise continuous and of exponential order.

9. True or False: $f(t) = e^{6t^2}$ is of exponential order.

10. True or False: If $L\{f(t)\} = F(s)$, then $L\{te^{8t} f(t)\} = F(s-8)$.

Part III (60 points): Show all steps in each problem to earn full credit.

11. Evaluate $L\{t\int_0^t \tau e^{-\tau} d\tau\}$

12. Write the given function in terms of the unit step function and find its Laplace transform.

$$f(t) = \begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$$

13. Use the definition $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ to find the Laplace transform of $f(t) = \sinh kt$.

14. Evaluate $f(t) = L^{-1}\left\{\dfrac{s}{(s-2)(s-3)(s-6)}\right\}$

15. Use the Laplace transform to solve the given initial value problem.

$$y'' + y = \delta(t - \tfrac{\pi}{2}) + \delta(t - \tfrac{3\pi}{2}), \quad y(0) = 0, \qquad y'(0) = 1$$

Bonus: Evaluate $L\{\cos^2 t\}$