MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the figure below to answer the following question(s).

1) Which of the following is a bridge of the graph?  
   A) BD  
   B) AD  
   C) CC  
   D) EC  
   E) none of these

Assume you have a graph with vertex set \( V = \{A, B, C, D, E\} \) and edge set \( E = \{CD, CB, DA, DB, EA, AA\} \).

2) How many paths are there from B to A?  
   A) 2  
   B) 4  
   C) 1  
   D) 0  
   E) none of these are true.

Use the figure below to answer the following question(s).

Graph 1

Graph 2

Graph 3

Graph 4

3) Which graphs are disconnected?  
   A) Graph 2 and Graph 3  
   B) Graph 2 only  
   C) Graph 3 only  
   D) Graph 1 and Graph 4  
   E) none of these
Use the figure below to answer the following question(s).

4) Which of the drawings has a closed unicursal tracing?
   A) Figure 2 only
   B) Figure 1 only
   C) Figure 1 and Figure 3
   D) Figure 3 only
   E) none of these

5) The number of vertices of odd degree in the graph that models this problem is
   A) 4.
   B) 8.
   C) 16.
   D) 12.
   E) none of these
In a certain city there is a river running through the middle of the city. There are three islands and seven bridges as shown in the figure below.

6) In the graph that models this situation, the degree of the vertex that represents island A is
   A) 2.
   B) 3.
   C) 4.
   D) 1.
   E) none of these

Use the figure below to answer the following question(s).

7) Which of the graphs has an Euler circuit?
   A) Graphs 1 and 3
   B) Graph 2 only
   C) Graph 1 only
   D) Graph 3 only
   E) none of these

Solve the problem.

8) A graph has twelve vertices—two vertices of degree 4, four vertices of degree 3, and six vertices of degree 2. The number of edges in the graph is
   A) 9.
   B) 14.
   C) 16.
   D) 12.
   E) none of these
Use the figure below to answer the following question(s).

9) Which of the graphs has an Euler path but no Euler circuit?
   A) Graphs 1 and 2
   B) Graph 3 only
   C) Graph 2 only
   D) Graph 1 only
   E) none of these

9) ________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

10) Find an Euler circuit for the following graph by labeling the edges 1, 2, 3, and so on in the order in which they can be traveled.

10) ____________

11) Find an Euler path for the following graph by labeling the edges 1, 2, 3, and so on in the order in which they can be traveled.

11) ____________
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

12) The basic rule in Fleury’s algorithm is
   A) never travel across a bridge of the untraveled part of the graph.
   B) only travel across a bridge of the untraveled part of the graph if there is no other alternative.
   C) only travel across a bridge on the original graph if there is no other alternative.
   D) never travel across a bridge of the original graph.
   E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

An undercover police officer is assigned the job of once a night walking each of the 48 blocks of a certain section of town described by the street grid shown below. The walk starts and ends at A. The officer wants to minimize the total number of blocks he has to walk each night.

![street grid diagram]

13) An optimal eulerization of the graph representing this problem can be obtained by adding how many edges? Draw each edge.

14) Suppose that it takes the officer 5 minutes to walk a block. In an optimal trip, the officer will cover the entire neighborhood in what amount of time?

Solve the problem.

15) After the eulerization of a graph, the number of odd vertices is

16) After the semi-eulerization of a graph, the number of odd vertices is
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

17) The following graph

A) has no Hamilton circuit.
B) has a single Hamilton circuit (and its mirror-image circuit).
C) has several Hamilton circuits, all of which contain the edge GC.
D) has several Hamilton circuits, none of which contain the edge BC.
E) none of these

18) n! =
A) $1 \times 2 \times 3 \times \ldots \times n$
B) $\frac{1}{2}n(n - 1)$
C) $1 + 2 + 3 + \ldots + n$
D) $n + 1$
E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

19) The number of Hamilton circuits in $K_{11}$ is

20) In a complete graph with 720 distinct Hamilton circuits, how many vertices are there?

21) In a complete graph with 12 vertices (A through L), the total number of Hamilton circuits (including mirror-image circuits) that start at vertex A is?

22) The number of edges in $K_{11}$ is?

A delivery truck must deliver furniture to 4 different locations (A, B, C, and D). The trip must start and end at A. The graph below shows the distances (in miles) between location. We want to minimize the total distance traveled.

23) The nearest-neighbor tour starting with vertex A is?

24) An optimal tour is?
25) The repetitive nearest-neighbor tour starting with vertex A is?

26) The cheapest-link tour starting with vertex A is given by:

A delivery truck must deliver packages to 6 different store locations (A, B, C, D, E, and F). The trip must start and end at A. The graph below shows the distances (in miles) between locations. We want to minimize the total distance traveled.

27) How many different Hamilton circuits would we have to check if we use the brute-force algorithm? (Do not count the same circuit traveled backward.)

28) The nearest-neighbor tour starting with vertex A is?

29) The repetitive nearest-neighbor tour starting with vertex A is?

30) The cheapest-link tour starting with vertex A is given by:

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

31) The brute-force algorithm for solving the Traveling Salesman Problem is
   A) an optimal and inefficient algorithm.
   B) an approximate and efficient algorithm.
   C) an optimal and efficient algorithm.
   D) an approximate and inefficient algorithm.
   E) none of these

32) The nearest-neighbor algorithm for solving the Traveling Salesman Problem is
   A) an optimal and inefficient algorithm.
   B) an approximate and efficient algorithm.
   C) an approximate and inefficient algorithm.
   D) an optimal and efficient algorithm.
   E) none of these

33) The repetitive nearest-neighbor algorithm for solving the Traveling Salesman Problem is
   A) an optimal and inefficient algorithm.
   B) an approximate and efficient algorithm.
   C) an approximate and inefficient algorithm.
   D) an optimal and efficient algorithm.
   E) none of these
34) The cheapest-link algorithm for solving the Traveling Salesman Problem is
   A) an optimal and efficient algorithm.
   B) an approximate and inefficient algorithm.
   C) an optimal and inefficient algorithm.
   D) an approximate and efficient algorithm.
   E) none of these

35) Which of the following four graphs is a tree?

   Graph 1

   A) Graph 1 and Graph 4
   B) Graph 2 and Graph 3
   C) Graph 2 and Graph 4
   D) Graph 1 and Graph 3
   E) none of these

36) A tree is
   A) any graph with one component.
   B) any graph that is connected and has no circuits.
   C) any graph that has no bridges.
   D) any graph that has no circuits.
   E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

37) The number of edges in a tree with 49 vertices is? 37) __________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

38) Assume that G is a graph with no loops or multiple edges. Which of the following graphs G are definitely trees?
   A) G has 14 vertices, 13 edges, and no circuits.
   B) G has 14 vertices, is connected, and every edge in G is a bridge.
   C) G has 14 vertices and there is exactly one path from any vertex to any other vertex.
   D) all of these are trees.
   E) none of these are trees.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

39) A vertex of degree 1 in a tree is often called a leaf. What is the smallest number of leaves that a tree on 100 vertices can have? 39) __________
40) How many spanning trees does the following network have? 

41) Calculate the redundancy of the following network. 

42) Find the MST of the network using Kruskal’s algorithm.

43) Give the weight of the MST.
The question(s) that follow refer to the problem of finding the minimum spanning tree for the weighted network shown below.

![Network Diagram]

44) Using Kruskal’s algorithm, which edge should we choose first?  
45) Using Kruskal’s algorithm, which edge should we choose third?  
46) Using Kruskal’s algorithm, which edge should we choose last?  
47) The total weight of the minimum spanning tree is 

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

48) Which of the following statements is true about Kruskal’s algorithm.  
   A) It is an efficient algorithm, but it doesn’t always give the minimum spanning tree.  
   B) It is an inefficient algorithm, but it always gives the minimum spanning tree.  
   C) It is an efficient algorithm, and it always gives the minimum spanning tree.  
   D) It is an inefficient algorithm, and it never gives the minimum spanning tree.  
   E) none of these 

49) The shortest network connecting the points A, B, and C shown below has 

![Network Diagram]

   A) a Steiner point inside the triangle ABC.  
   B) no junction point.  
   C) a Steiner point outside the triangle ABC.  
   D) a junction point at A.  
   E) none of these
Assume you have four cities (A, B, C, and D) located at the corners of a 100 mile by 100 mile square which are to be connected by a network of power lines.

50) Which of the following figures represents the shortest network of power lines connecting these four cities?

A) 

B) 

C) 

D) 

E) none of these

Solve the problem.

51) One of the four points below – W, X, Y, or Z – is a Steiner point of triangle ABC. Which one?

A) Z
B) X
C) W
D) Y
E) all of these
52) The shortest network connecting a set of points
A) is either a Steiner tree or a minimum spanning tree.
B) is always shorter than a minimum spanning tree.
C) is always a minimum spanning tree.
D) is always a Steiner tree.
E) none of these

53) The number of Steiner points in a shortest network connecting four cities can only be
A) 1.
B) 2.
C) either 0, 1, or 2.
D) either 1 or 2.
E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

For the following question(s), refer to the digraph below.

54) Vertex B has what indegree? What outdegree?

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

55) Which of the following is not a path from vertex C to vertex E in the digraph?
A) all of these are paths from C to E.
B) C, D, E
C) C, A, E
D) C, A, B, E
E) none of these are paths from C to E.

56) Which of the following is not a cycle in the digraph?
A) A, E, B, A
B) A, B, A, E, C, A
C) E, C, D, E
D) A, E, C, A
E) all of these are cycles in the digraph.
57) Suppose that the vertices of the digraph represent individuals and there is an arc going from vertex X to vertex Y if and only if X "likes" Y. Which of the following statements [A), B), C), or D)] is not true?
   A) A likes B and E but does not like C or D.
   B) B and C do not like each other.
   C) A and B like each other.
   D) B likes E but E does not like B.
   E) all of these statements are true.

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

For the following question(s), refer to the tournament digraph below. The vertices of the digraph represent five tennis players in a round-robin tournament. An arc XY represents the fact that X defeated Y in the tournament.

58) Which player won the tournament?  
59) How many games were played in the tournament?
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Suppose you have the project consisting of the six tasks described in the following table.

<table>
<thead>
<tr>
<th>Task</th>
<th>Length of task (hours)</th>
<th>Prior tasks that must be completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>A, E</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>F</td>
<td>8</td>
<td>B</td>
</tr>
</tbody>
</table>

60) Which project digraph below models the project described?

A) [Diagram A]

B) [Diagram B]

C) [Diagram C]

D) [Diagram D]

E) none of these

60) _______
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Suppose you have the following project digraph. (The numbers in parentheses represent hours.)

![Project Digraph]

61) The number of tasks in the project is? 61) ___________

62) Using the priority list, C, F, E, B, A, D and the priority–list model to schedule this project with two processors results in a finishing time of 62) ___________

Solve the problem.

63) What is the number of possible priority lists in a project with 8 tasks? 63) ___________

Suppose you have the following project digraph. (The numbers in parentheses represent hours.)

![Project Digraph]

64) Using the decreasing–time algorithm to schedule this project with two processors results in a completion time of 64) ___________
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Suppose you have the following project digraph. (The numbers in parentheses represent hours.)

65) If we use decreasing-time algorithm to schedule this project with two processors, we should start by assigning  
   A) task B to one processor, task C to the other one.  
   B) task A to one processor, task C to the other one.  
   C) task A to one processor, task B to the other one.  
   D) task B to one processor, task E to the other one.  
   E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

66) Using the decreasing-time algorithm to schedule this project with two processors, the project finishing time is

67) Using the decreasing-time algorithm to schedule this project with two processors, the total combined idle time of the two processors is

Suppose you have the following project digraph. (The numbers in parentheses represent hours.)

68) The length of the critical path from C is

69) The length of the critical path from B is

70) The length of the critical path for the entire project is

71) Using critical-path algorithm to schedule this project with two processors results in a finishing time of
72) Using the critical-path algorithm to schedule this project with six processors results in a finishing time of

A project consists of seven independent tasks with processing times (in hours) given by 4, 5, 6, 7, 8, 9, and 12.

73) Find the project finishing time $F_{in}$ for $N = 2$ processors using the critical-path algorithm.

74) Find the optimal finishing time $O_{pt}$ for $N = 2$ processors.

75) Compute the relative error of the critical-path schedule expressed as a percent.

Suppose you have the following project digraph. (The numbers in parentheses represent hours.)

76) The optimal finishing time for this project using two processors is

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

77) Which of the following is a priority list that would produce the timeline of independent tasks shown below?

<table>
<thead>
<tr>
<th>Time:</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>G(4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_2$</td>
<td>N(8)</td>
<td>D(3)</td>
<td>I(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_4$</td>
<td>A(3)</td>
<td>C(4)</td>
<td>E(2)</td>
<td>K(4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_5$</td>
<td>L(5)</td>
<td>F(3)</td>
<td>H(4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Finishing time = 13

C) A, B, C, D, E, F, G, H, I, J, K, L, M, N
E) none of these
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

78) Find a 4-coloring for the graph below.

79) Find $\chi(G)$ for the graph below.
80) Give a coloring for the four states below and use the fewest possible colors.

81) Remember that there will be several questions from the Politics half, so you should read over and rework your midterm review, as well.
Answer Key
Testname: FINALREVIEW

1) A
2) B
3) A
4) A
5) C
6) A
7) D
8) C
9) D
10) 

11) 

12) B
13) 12 edges.
14) 5 hours.
15) 0.
16) 2.
17) A
18) A
19) 10!
20) 7 vertices.
21) 11!
22) 55.
23) A, D, C, B, A.
24) A, B, D, C, A.
25) A, B, D, C, A
26) A, B, D, C, A.
27) 60
28) A, D, F, E, C, B, A.
29) A, E, F, C, B, D, A.
30) A, E, F, C, B, D, A.
31) A
Answer Key
Testname: FINALREVIEW

32) B
33) B
34) D
35) D
36) B
37) 48.
38) B
39) 2
40) 3
41) 10
42)

43) 89
44) BE
45) EF
46) AC
47) 55.
48) C
49) A
50) C
51) D
52) A
53) C
54) indegree 2, outdegree 1.
55) D
56) E
57) D
58) D
59) 10
60) C
61) 6.
62) 22 hours.
63) 8!
64) 22 hours.
65) B
66) 20 hours.
67) 7 hours.
68) 15 hours.
69) 15 hours.
70) 18 hours.
Answer Key
Testname: FINALREVIEW

71) 19 hours.
72) 18 hours.
73) Fin = 27 hours
74) Opt = 26 hours
75) 3.8%
76) 19 hours.
77) B
78) A, C, F: red; B, D, G: green; E, H: blue; I: yellow
79) 4
80) State A: red; State B: blue; State C: green; State D: yellow
81)